

# Quantum Monte Carlo calculations of magnetic moments and M1 transitions in $A \leq 9$ nuclei

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**Abstract.** We present Quantum Monte Carlo calculations of magnetic moments and M1 transitions in  $A \leq 9$  nuclei which take into account contributions of two-body electromagnetic currents. The Hamiltonian utilized to generate the nuclear wave functions includes the realistic Argonne- $v_{18}$  two-nucleon and the Illinois-7 three-nucleon interactions. The nuclear two-body electromagnetic currents are derived from a pionful chiral effective field theory including up to one-loop corrections. These currents involve unknown Low Energy Constants which have been fixed so as to reproduce a number of experimental data for the two- and three-nucleon systems, such as  $np$  phase shifts and deuteron, triton, and  ${}^3\text{He}$  magnetic moments. This preliminary study shows that two-body contributions provide significant corrections which are crucial to bring the theory in agreement with the experimental data in both magnetic moments and M1 transitions.

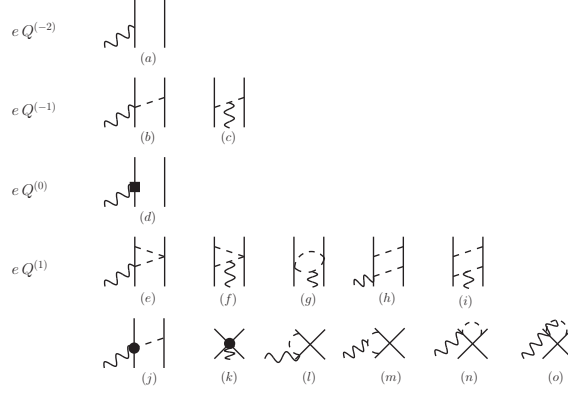
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## INTRODUCTION

Quantum Monte Carlo calculations of magnetic moments (m.m.'s) and M1 transitions in  $A \leq 7$  nuclei have been presented in recent years by Marcucci *et al.* in Ref. [1]. In that work, the authors investigated the role played by the electromagnetic (EM) two-body meson-exchange currents (MEC) derived from the realistic nucleon-nucleon (NN) interaction Argonne  $v_{18}$  (AV18) via current conservation [1, 2]. It was found that MEC contributions increase the  $A = 3, 7$  isovector m.m.'s, as well as the  $A = 6, 7$  M1 transitions by 16% and 17 – 34%, respectively, bringing them into very good agreement with experimental data. In this framework—also referred to as the Standard Nuclear Physics Approach (SNPA)—the isoscalar m.m.'s of the  $A = 3, 7$  nuclei are, however, underpredicted by a few percent (up to 10% in  $A = 7$  systems) [1].

Two-body EM current operators have been derived recently within pionful chiral effective field ( $\chi$ EFT) formulations [3, 4, 5, 6, 7, 8]. The  $\chi$ EFT current operators are expanded in powers of pions' and nucleons' momenta, and consist of long- and intermediate-range components which are described in terms of one- and two-pion exchange contributions, as well as contact currents which encode the short-range physics. These operators involve a number of Low Energy Constants (LECs) which are then fixed to the experimental data. In hybrid calculations, many-body operators derived from a  $\chi$ EFT framework, are utilized in transition matrix elements in between nuclear wave functions (w.f.'s) obtained from Hamiltonians involving realistic two- and three-body nuclear interactions. Intrinsic to this approach is a mismatch between the short-range



**FIGURE 1.** Diagrams illustrating one- and two-body EM currents entering at LO ( $eQ^{-2}$ ), NLO ( $eQ^{-1}$ ), N2LO ( $eQ^0$ ), and N3LO ( $eQ^1$ ). Nucleons, pions, and photons are denoted by solid, dashed, and wavy lines, respectively.

behavior of the NN potential and that of the current operators. Hybrid calculations of EM observables in the  $A = 2-4$  nuclei (see for example Refs. [9, 10, 11, 12, 13]) indicate that this inconsistency is in most cases mitigated by the fitting procedure implemented to constrain the LECs.

The analysis presented here has two objectives. The first one is to extend the studies reported in Ref. [1] to  $A > 7$  nuclei. The second one is to investigate how the  $\chi$ EFT EM current operators of Refs. [4, 5], *albeit* utilized within a hybrid context, compare with the SNPA formulation of Ref. [1]. In what follows we briefly report on the Quantum Monte Carlo (QMC) techniques, and the  $\chi$ EFT EM operators utilized in these calculations, and provide preliminary results obtained from them.

## QMC METHOD AND THE NUCLEAR HAMILTONIAN

The EM transition matrix elements are evaluated in between w.f.'s which are solutions of the Schrödinger equation

$$H|\Psi\rangle = E|\Psi\rangle . \quad (1)$$

The nuclear Hamiltonian used in the calculations consists of a kinetic term plus two- and three-body interaction terms, namely the AV18 [14] and the Illinois-7 [15], respectively:

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} . \quad (2)$$

Nuclear w.f.'s are constructed in two steps. First, a trial variational Monte Carlo w.f. ( $\Psi_T$ ), which accounts for the effect of the nuclear interaction via the inclusion of correlation operators, is generated by minimizing the energy expectation value with respect to a number of variational parameters. The second step improves on  $\Psi_T$  by eliminating excited states contamination. This is accomplished in a Green's function Monte Carlo (GFMC) calculation which propagates the Schrödinger equation in imaginary time ( $\tau$ ). The propagated w.f.  $\Psi(\tau) = e^{-(H-E_0)\tau}\Psi_T$ , for large values of  $\tau$ , converges to the exact w.f. with eigenvalue  $E_0$ . Ideally, the matrix elements should be evaluated in between

two propagated w.f.'s. In practice, we evaluate mixed estimates in which only one w.f. is propagated, while the remaining one is replaced by  $\Psi_T$ . The calculation of diagonal and off-diagonal matrix elements is discussed at length in Ref. [16] and references therein.

The nuclear EM current operator—regardless of the formalism utilized to construct it—is also expressed as an expansion in many-body operators. The current utilized in this work accounts up to two-body effects, and is written as:

$$\mathbf{j}(\mathbf{q}) = \sum_i \mathbf{j}_i(\mathbf{q}) + \sum_{i<j} \mathbf{j}_{ij}(\mathbf{q}) , \quad (3)$$

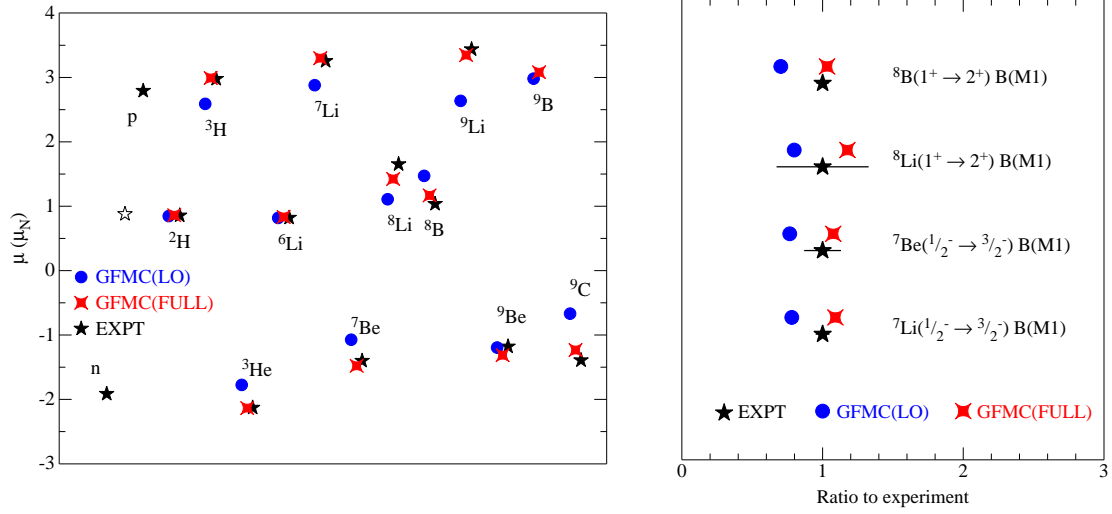
where  $\mathbf{q}$  is the momentum associated with the external EM field.

## EM CURRENTS AND MAGNETIC MOMENTS IN $\chi$ EFT

Currents from pionful  $\chi$ EFT including up to two-pion exchange contributions were derived originally by Park, Min, and Rho in covariant perturbation theory [3]. More recently, Kölling and collaborators presented EM currents obtained within the method of unitary transformations [7, 8]. Here, we refer to the EM operators constructed in Ref. [5], in which time-ordered perturbation theory is implemented to calculate the EM transition amplitudes. Differences between the models mentioned above have been discussed in Refs. [5, 8], and will not be addressed here. Instead, we limit ourselves to briefly describing the various contributions to the EM currents which have been utilized in the GFMC calculations, and refer to Ref. [5] for the formal expressions of the operators. However, we remark that we have revised our calculation [17] of the operators associated with diagrams (m) and (o) of Fig. 1.

The  $\chi$ EFT EM current operators are diagrammatically represented in Fig. 1. They are expressed as an expansion in  $Q$ , *i.e.*, the low-momentum scale. Referring to Fig. 1, the leading-order (LO) term is counted as  $eQ^{-2}$  ( $e$  is the electric charge), and consists of the single-nucleon convection and spin-magnetization currents. The NLO term (of order  $eQ^{-1}$ ) involves seagull and in-flight contributions associated with one-pion exchange, and the N2LO term (of order  $eQ^0$ ) represents the  $(Q/m_N)^2$  relativistic correction to the LO one-body current ( $m_N$  denotes the nucleon mass). At N3LO ( $eQ$ ) we include one-loop contributions of diagrams (e)–(i) and (l)–(o), as well as the tree-level current involving a  $\gamma\pi NN$  vertex of order  $eQ^2$ —of diagram (j), and the contact currents of diagram (k). The two-body operators have a power-law behavior at large momenta, therefore a regularization procedure is implemented via the introduction of cutoff function of the form  $\exp(-Q^4/\Lambda^4)$  [17], where  $\Lambda = 600$  MeV.

The contact currents of diagram (k) involve both minimal and non-minimal LECs. Minimal LECs enter the  $\chi$ EFT contact NN interaction at order  $Q^2$ , and can be taken from fits to the NN scattering data. We use the values obtained from the analysis of Refs. [18, 19], with cutoff  $\Lambda = 600$  MeV. Non-minimal LECs entering the contact and tree-level currents at N3LO—diagrams (j) and (k), respectively—need to be fixed to EM observables. The contact currents involve two LECs, multiplying an isoscalar and an isovector operator, respectively. There are three LECs entering the tree-level current of diagram (j). Two of them multiply isovector structures and they saturate the  $\Delta$ -resonance excitation current, while the third one is associated with an isoscalar operator and saturates the  $\rho\pi\gamma$  transition current [3]. We exploit the  $\Delta$ -resonance saturation mechanism,



**FIGURE 2.** **Left:** Magnetic moments in nuclear magnetons for  $A \leq 9$  nuclei. Black stars indicate the experimental values [20, 21], while blue dots (red diamonds) represent preliminary GFMC calculations which include the LO one-body EM current (full  $\chi$ EFT current up to N3LO). Predictions are for nuclei with  $A > 3$ . **Right:** Transition widths normalized to the experimental values [20, 21] for  $A = 7-8$  nuclei, notation as in left panel.

thus reducing the number of unknown LECs to three. We fix the two isoscalar LECs so as to reproduce the deuteron and the isoscalar combination of the trinucleon m.m.'s, while the isovector LEC is obtained from fits to the isovector combination of the  $A = 3$  nuclei m.m.'s. This choice provides us with the most natural LECs [17].

## RESULTS

The preliminary results for the m.m.'s of  $A \leq 9$  nuclei are summarized in the left panel of Fig. 2. In this figure, black stars represent the experimental data [20, 21]—there are no data for the m.m. of  $^9\text{B}$ . For completeness, we show also the experimental values for the proton and neutron m.m.'s, as well as their sum, which corresponds to the m.m. of an S-wave deuteron. The experimental values of the  $A = 2-3$  m.m.'s have been utilized to fix the LECs, therefore predictions are for  $A > 3$  nuclei. The blue dots labeled as GFMCLO represent theoretical predictions obtained with the standard one-nucleon EM current entering at LO—diagram a) of Fig. 1. The GFMCLO results reproduce the bulk properties of the m.m.'s of the light nuclei considered here. In particular, we can recognize three classes of nuclei, that is nuclei whose m.m.'s are driven by an unpaired valence proton, or neutron, or ‘deuteron cluster’ inside the nucleus. Predictions which include all the contributions to the N3LO  $\chi$ EFT EM currents illustrated in Fig. 1 are represented by the red diamonds of Fig. 2, labeled GFMC(FULL). In most of the cases considered here, the predicted m.m.'s are closer to the experimental data when the corrections entering at NLO and following orders are added to the LO one-body EM operator. Notable are the cases associated with the  $A = 9$  and  $T = 3/2$  nuclei, in which these corrections are found to provide up to  $\sim 40\%$  of the total predictions. We

also tested how the SNPA currents perform for these EM observables and found that the hybrid  $\chi$ EFT formulation provides us with improved values for the isoscalar m.m.'s of the nuclei considered here.

In the right panel of Fig. 2, we show the preliminary result for M1 transitions in  $A \leq 8$  nuclei. Here, we show the ratios to the experimental values of the widths [20, 21]. The latter are represented with the black stars along with the associated experimental error bars, while the GFMC(LO) and GFMC(FULL) predictions are again represented by blue dots and red diamonds, respectively. Also for these EM observables, predictions which account for the complete N3LO operator are closer to the experimental values, but for the transition in  $^8\text{Li}$ , for which the experimental error is large, we cannot determine whether the GFMC(FULL) prediction is a better one. The study presented here is at its first stage. A manuscript with a detailed presentation of this work is in preparation [22].

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## REFERENCES

1. L. E. Marcucci, M. Pervin, S. C. Pieper, R. Schiavilla, and R. B. Wiringa, *Phys. Rev. C* **78**, 065501 (2008).
2. L. E. Marcucci, M. Viviani, R. Schiavilla, A. Kievsky, and S. Rosati, *Phys. Rev. C* **72**, 014001 (2005).
3. T.-S. Park, D.-P. Min, and M. Rho, *Nuclear Physics A* **596**, 515 (1996).
4. S. Pastore, R. Schiavilla, and J. L. Goity, *Phys. Rev. C* **78**, 064002 (2008).
5. S. Pastore, L. Girlanda, R. Schiavilla, M. Viviani, and R. B. Wiringa, *Phys. Rev. C* **80**, 034004 (2009).
6. S. Pastore, L. Girlanda, R. Schiavilla, and M. Viviani, *Phys. Rev. C* **84**, 024001 (2011).
7. S. Kölling, E. Epelbaum, H. Krebs, and U. G. Meißner, *Phys. Rev. C* **80**, 045502 (2009).
8. S. Kölling, E. Epelbaum, H. Krebs, and U.-G. Meißner, *Phys. Rev. C* **84**, 054008 (2011).
9. T.-S. Park, L. E. Marcucci, R. Schiavilla, M. Viviani, A. Kievsky, S. Rosati, K. Kubodera, D.-P. Min, and M. Rho, *Phys. Rev. C* **67**, 055206 (2003).
10. Y.-H. Song, R. Lazauskas, T.-S. Park, and D.-P. Min, *Physics Letters B* **656**, 174 (2007).
11. Y.-H. Song, R. Lazauskas, and T.-S. Park, *Phys. Rev. C* **79**, 064002 (2009).
12. R. Lazauskas, Y.-H. Song, and T.-S. Park, *Phys. Rev. C* **83**, 034006 (2011).
13. L. Girlanda, A. Kievsky, L. E. Marcucci, S. Pastore, R. Schiavilla, and M. Viviani, *Phys. Rev. Lett.* **105**, 232502 (2010).
14. R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, *Phys. Rev. C* **51**, 38 (1995).
15. S. C. Pieper, *AIP Conference Proceedings* **1011**, 143 (2008).
16. M. Pervin, S. C. Pieper, and R. B. Wiringa, *Phys. Rev. C* **76**, 064319 (2007).
17. L. Girlanda, S. Pastore, R. Schiavilla, and M. Viviani (in preparation).
18. D. R. Entem, and R. Machleidt, *Phys. Rev. C* **68**, 041001 (2003).
19. R. Machleidt, and D. Entem, *Physics Reports* **503**, 1 (2011).
20. D. Tilley, C. Cheves, J. Godwin, G. Hale, H. Hofmann, J. Kelley, C. Sheu, and H. Weller, *Nuclear Physics A* **708**, 3 (2002).
21. D. Tilley, J. Kelley, J. Godwin, D. Millener, J. Purcell, C. Sheu, and H. Weller, *Nuclear Physics A* **745**, 155 (2004).
22. S. Pastore, S. C. Pieper, R. Schiavilla, and R. B. Wiringa (in preparation).